From Petri nets to higher-dimensional automata

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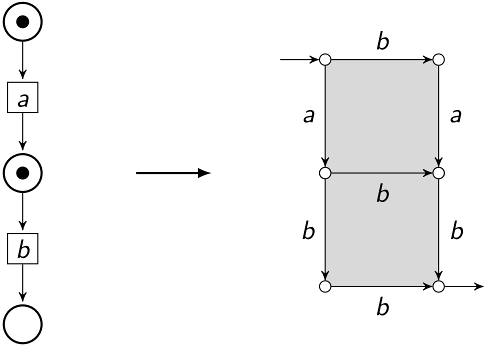
Overview

1. Introduction
2. Definitions
3. Algorithm and Implementation

Project goal



Project overview

- Load Petri nets from files
- Implement the conversion algorithm
- Check the behavior of Petri net and convertised HDA

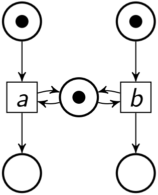
Petri nets (PN)

Definition

$$PN = (S, T, F, M_0, l)$$

We have:

- $S \cap T = \emptyset$
- $F : (S \times T \cup T \times S) \to \mathbb{N}$
- $M_0 : S \to \mathbb{N}$
- $l : T \to A$ (with A a set of actions).



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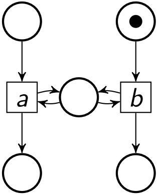

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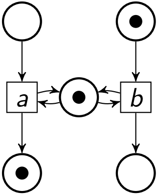

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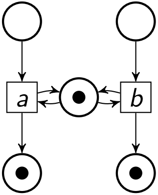
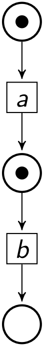


ST-Traces of Petri nets



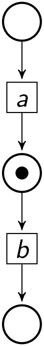
valid examples:

$$\begin{cases} a^+ b^+ a^- b^+ b^- b^- \\ b^+ b^- a^+ a^- b^+ b^- \\ a^+ b^+ b^- a^- b^+ b^- \\ \ldots \end{cases}$$

invalid example:

$$b^+ b^+ b^- b^- a^+ a^-$$

ST-Traces of Petri nets



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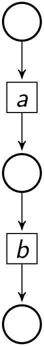

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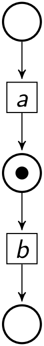

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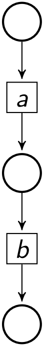

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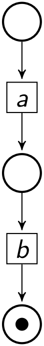

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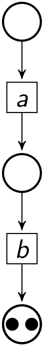

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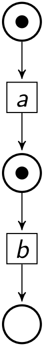

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invalid example:

$$b^+ b^+ b^- b^- a^+ a^-$$

Pomsets

Definition

$$P = (E, \prec, \lambda)$$

$$\left(\begin{tikzcd} & c \\ a \arrow[ur] \arrow[r] & b \end{tikzcd}\right)$$

$$a \prec c \wedge a \prec b$$
$$\neg(c \prec b) \wedge \neg(b \prec c)$$

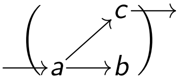
IPomsets

Definition

$$P = (E, \prec, \lambda, S, T)$$

where:

- $(E, \prec, \lambda)$ is a Pomset
- $S \subseteq E$ and $T \subseteq E$



$$S = \{a\}$$
$$T = \{c\}$$

Gluing Operation

Definition

$$P * Q = (E_P \cup E_Q, \prec, \lambda, S_P, T_Q)$$

with

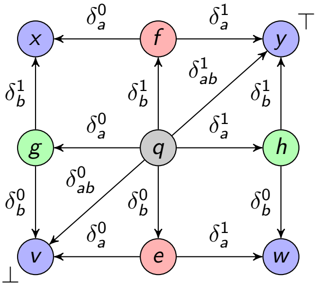
- $x \prec y$ if $x \prec_P y$ or $x \prec_Q y$ or $(x \in P \setminus T_P) \wedge (y \in Q \setminus S_Q)$
- $\lambda(x) = \begin{cases} \lambda_P(x) & \text{if} \quad x \in P \\ \lambda_Q(x) & \text{if} \quad x \in Q \end{cases}$

$$\left(\begin{tikzcd} & a \arrow[r] & {} \\ b \arrow[r] & c & \end{tikzcd}\right) * \left(\begin{tikzcd} {} \arrow[r] & a \arrow[r] & e \\ & d & \end{tikzcd}\right) = \left(\begin{tikzcd} & a \arrow[r] & e \\ b \arrow[r] & c \arrow[ur] \arrow[r] & d \end{tikzcd}\right)$$

Precubical set

Definition

$$X = \{X, ev, \{\delta^0_{A,U}, \delta^1_{A,U} | A \subseteq U,\ U \in \square\}\}$$

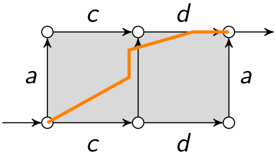


Higher-Dimensional Automata (HDA)

Definition

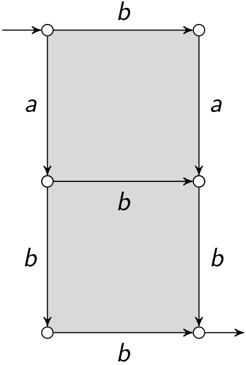
$$HDA = (X, \bot_X, \top_X)$$

where $\bot_X \subseteq X$ and $\top_X \subseteq X$



$$P = \begin{pmatrix} a & \\ c \longrightarrow d \end{pmatrix}$$

Language of HDA



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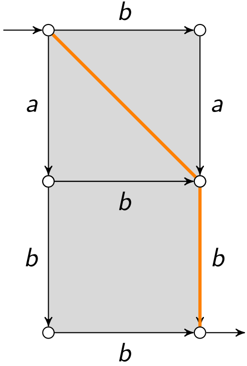
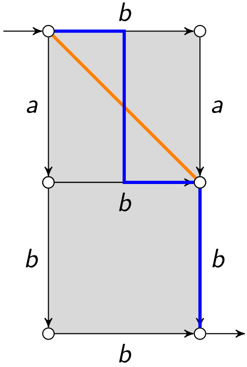


$$P = \left(\begin{tikzcd} b \arrow[dr] & \\ a \arrow[r] & b \end{tikzcd}\right) = \begin{pmatrix} b \\ \\ a \end{pmatrix} * \begin{pmatrix} b \end{pmatrix}$$

Language of HDA



$$\boxed{b^{+} a^{+} a^{-} b^{-} b^{+} b^{-}}$$

$$P = \left(\begin{tikzcd} b \arrow[dr] & \\ a \arrow[r] & b \end{tikzcd}\right) = \begin{pmatrix} b \\ a \end{pmatrix} * \begin{pmatrix} b \end{pmatrix}$$

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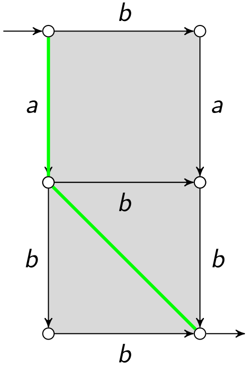


$$P = \left(\begin{tikzcd}[row sep=tiny] & b \\ a \arrow[ur] \arrow[r] & b \end{tikzcd} \right)$$

From Pomsets to traces

Definition

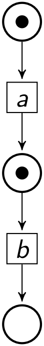
Let H be an HDA resulting from the conversion of the Petri net PN:

$$P \in \mathcal{L}(H) \iff ST(P) \subseteq ST(PN)$$

$$P = \left(\begin{tikzcd} b \arrow[dr] & \\ a \arrow[r] & c \end{tikzcd}\right)$$

$$ST(P) = \begin{Bmatrix} a^+ b^+ a^- b^- c^+ c^- \\ b^+ a^+ a^- b^- c^+ c^- \\ a^+ b^+ b^- a^- c^+ c^- \\ b^+ a^+ b^- a^- c^+ c^- \end{Bmatrix}$$

Conversion Algorithm



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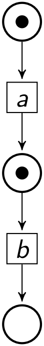
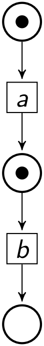


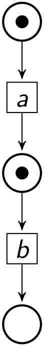
$$(1,1,0) \xrightarrow{b^{+}} (1,0,0) \xrightarrow{b^{-}} (1,0,1)$$

Conversion Algorithm



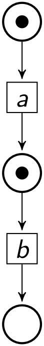
$$\begin{tikzcd}
(1,1,0) \arrow[r, "b^+"] \arrow[d, "a^+"'] & (1,0,0) \arrow[r, "b^-"] & (1,0,1) \\
(0,1,0) \arrow[d, "a^-"'] & & \\
(0,2,0) & &
\end{tikzcd}$$

Conversion Algorithm



$$\begin{tikzcd}
(1,1,0) \arrow[r, "b^{+}"] \arrow[d, "a^{+}"'] & (1,0,0) \arrow[r, "b^{-}"] & (1,0,1) \\
(0,1,0) \arrow[r, "b^{+}"] \arrow[d, "a^{-}"'] & (0,0,0) \arrow[r, "b^{-}"] & (0,0,1) \\
(0,2,0) & &
\end{tikzcd}$$

Conversion Algorithm



$$\begin{tikzcd}
(1,1,0) \arrow[r, "b^{+}"] \arrow[d, swap, "a^{+}"] & (1,0,0) \arrow[r, "b^{-}"] \arrow[d, swap, "a^{+}"] & (1,0,1) \arrow[d, swap, "a^{+}"] \\
(0,1,0) \arrow[r, "b^{+}"] \arrow[d, swap, "a^{-}"] & (0,0,0) \arrow[r, "b^{-}"] \arrow[d, swap, "a^{-}"] & (0,0,1) \arrow[d, swap, "a^{-}"] \\
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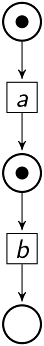

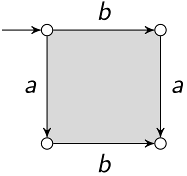

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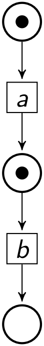
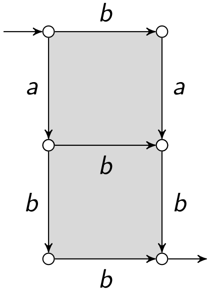
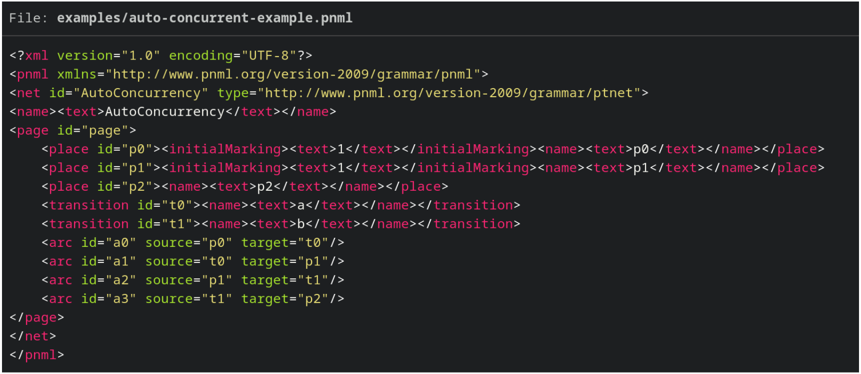




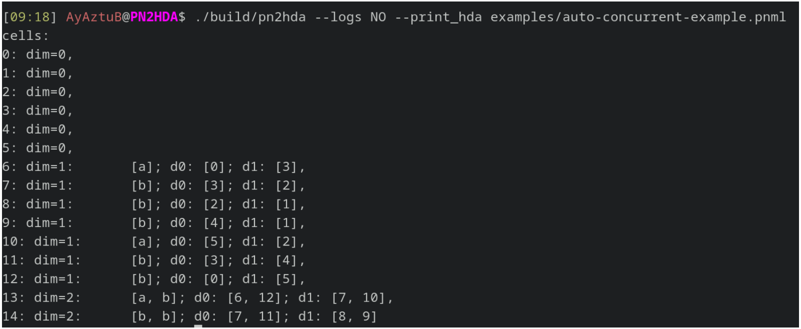
PNML file



File: **examples/auto-concurrent-example.pnml**

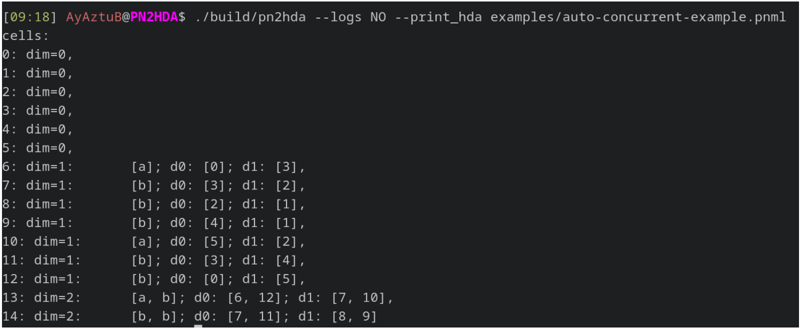
```xml
<?xml version="1.0" encoding="UTF-8"?>
<pnml xmlns="http://www.pnml.org/version-2009/grammar/pnml">
<net id="AutoConcurrency" type="http://www.pnml.org/version-2009/grammar/ptnet">
<name><text>AutoConcurrency</text></name>
<page id="page">
    <place id="p0"><initialMarking><text>1</text></initialMarking><name><text>p0</text></name></place>
    <place id="p1"><initialMarking><text>1</text></initialMarking><name><text>p1</text></name></place>
    <place id="p2"><name><text>p2</text></name></place>
    <transition id="t0"><name><text>a</text></name></transition>
    <transition id="t1"><name><text>b</text></name></transition>
    <arc id="a0" source="p0" target="t0"/>
    <arc id="a1" source="t0" target="p1"/>
    <arc id="a2" source="p1" target="t1"/>
    <arc id="a3" source="t1" target="p2"/>
</page>
</net>
</pnml>
```

Conversion output



```
[09:18] AyAztuB@PN2HDA$ ./build/pn2hda --logs NO --print_hda examples/auto-concurrent-example.pnml
cells:
0: dim=0,
1: dim=0,
2: dim=0,
3: dim=0,
4: dim=0,
5: dim=0,
6: dim=1:       [a]; d0: [0]; d1: [3],
7: dim=1:       [b]; d0: [3]; d1: [2],
8: dim=1:       [b]; d0: [2]; d1: [1],
9: dim=1:       [b]; d0: [4]; d1: [1],
10: dim=1:      [a]; d0: [5]; d1: [2],
11: dim=1:      [b]; d0: [3]; d1: [4],
12: dim=1:      [b]; d0: [0]; d1: [5],
13: dim=2:      [a, b]; d0: [6, 12]; d1: [7, 10],
14: dim=2:      [b, b]; d0: [7, 11]; d1: [8, 9]
```

Conversion output



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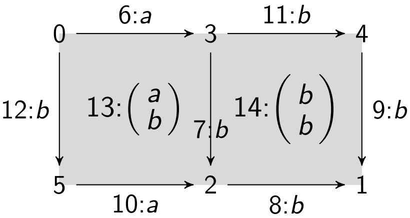
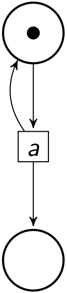

6:a
11:b
0
3
4
12:b
13:$\begin{pmatrix} a \\ b \end{pmatrix}$
7:b
14:$\begin{pmatrix} b \\ b \end{pmatrix}$
9:b
5
2
1
10:a
8:b

Implementation

- Language C
- Generate all possible executions
- Complexity exponential ($\mathcal{O}(3^n)$)

Unbounded Petri nets

- Always a resource in the first place
- Infinite number of activation of transition *a*
- Infinitely many configurations



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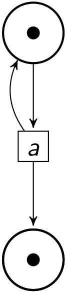

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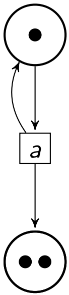

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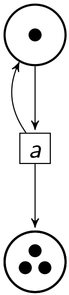

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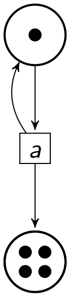

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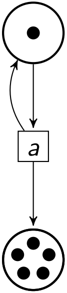

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